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Ishikawa and Mann Iterative Processes with Errors for Nonlinear Strongly Accretive Operator Equations*

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The purpose of this paper is to revise the definitions of Ishikawa and Mann iterative processes with errors, by using a new inequality to study the unique solution of the nonlinear strongly accretive operator equation $Tx = f$ and the convergence problem of Ishikawa and Mann iterative sequences for strongly pseudo-contractive mappings without the Lipschitz condition. The results presented in this paper improve and extend the corresponding results in [4, 5, 7–10, 12, 15, 16] in the more general setting. In particular, the open problem mentioned by Chidume in [5] has been given an affirmative answer. © 1998 Academic Press

Key Words: duality mapping; uniformly smooth Banach space; Mann iteration sequence; Ishikawa iteration sequence; strongly accretive mapping; strongly pseudo-contractive mapping.

1. INTRODUCTION AND PRELIMINARIES

Throughout this paper, we assume that X is a real Banach space and X^* is the dual space of X, $(\cdot, \cdot)$ denotes the pairing of X and X^*. The mapping $J\colon X \to 2^{X^*}$ defined by

$$J(x) = \{ j \in X^* : (x, j) = \|x\| \, \|j\|, \|j\| = \|x\| \} \tag{1.1}$$

is called the normalized duality mapping.

DEFINITION 1. Let X be a real normed space and K a nonempty subset of X.

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(i) A mapping $T\colon K \to X$ is said to be strongly accretive if for any x,y in K there exists a $j \in J(x-y)$ such that $(Tx - Ty, j) \geq k\|x-y\|^2$ for some constant $k > 0$. Without loss of generality we can assume that $k \in (0,1)$.

(ii) A mapping $T\colon X \to X$ is said to be strongly pseudo-contractive if $(I - T)$ (where I is the identity mapping on X) is strongly accretive.

The concept of accretive mapping was introduced independently by Browder [2] and Kato [11] in 1967. An early fundamental result in the theory of accretive mapping, due to Browder, states that the initial value problem

$$\frac{du(t)}{dt} + Tu(t) = 0, \qquad u(0) = u_0,$$

is solvable if T is locally Lipschitzian and accretive on X.

We first recall the following two iterative processes due to Ishikawa [10] and Mann [13], respectively.

(I) Let K be a nonempty convex subset of X and let $T\colon K \to K$ be a mapping. For any given $x_0 \in K$ the sequence $\{x_n\}$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Ty_n, \qquad y_n = (1-\beta_n)x_n + \beta_n Tx_n, \qquad n \geq 0,$$

is called the Ishikawa iteration sequence, where $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0,1]$ satisfying some conditions.

(I′) In particular, if $\beta_n = 0$ for all $n \geq 0$, then $\{x_n\}$ defined by

$$x_0 \in K, \qquad x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n, \qquad n \geq 0,$$

is called the Mann iteration sequence.

Recently, Liu introduced the concepts of Ishikawa and Mann iterative processes with errors for nonlinear strongly accretive mappings in [12] as follows.

(II) For a nonempty subset K of a Banach space X and a mapping $T\colon K \to X$, the sequence $\{x_n\}$ defined by

$$\begin{aligned} x_0 &\in K,\\ x_{n+1} &= (1-\alpha_n)x_n + \alpha_n Ty_n + u_n,\\ y_n &= (1-\beta_n)x_n + \beta_n Tx_n + v_n, \qquad n \geq 0, \end{aligned}$$

is called the Ishikawa iteration sequence with errors. Here $\{u_n\}$ and $\{v_n\}$ are two summable sequences in X (i.e., $\sum_{n=0}^{\infty}\|u_n\| < +\infty$ and $\sum_{n=0}^{\infty}\|v_n\| <$

$+\infty$), and $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in $[0,1]$ satisfying certain restrictions.

(II′) In particular, if $\beta_n = 0$ and $v_n = 0$ for all $n \geq 0$, the sequence $\{x_n\}$ defined by

$$x_0 \in K, \qquad x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T x_n + u_n, \qquad n \geq 0,$$

is called the Mann iteration sequence with errors. Here $\{u_n\}$ is a summable sequence in X, and $\{\alpha_n\}$ is a sequence in $[0,1]$ satisfying certain restrictions.

Unfortunately, the definitions of Liu, which depend on the convergence of the error terms, is against the randomness of errors. Hence, we need a new definition as follows.

(III) Let K be a nonempty convex subset of X and let $T: K \to K$ be a mapping. For any given $x_0 \in K$ the sequence $\{x_n\}$ defined by

$$\begin{aligned} x_{n+1} &= \alpha_n x_n + \beta_n T y_n + \gamma_n u_n, \\ y_n &= \hat{\alpha}_n x_n + \hat{\beta}_n T x_n + \hat{\gamma}_n v_n, \qquad n \geq 0, \end{aligned} \tag{1.2}$$

is called the Ishikawa iteration sequence with errors. Here $\{u_n\}$ and $\{v_n\}$ are two bounded sequences in K; $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\hat{\alpha}_n\}$, $\{\hat{\beta}_n\}$, and $\{\hat{\gamma}_n\}$ are six sequences in $[0,1]$ satisfying the conditions

$$\alpha_n + \beta_n + \gamma_n = \hat{\alpha}_n + \hat{\beta}_n + \hat{\gamma}_n = 1 \qquad \text{for all } n \geq 0. \tag{1.3}$$

(III′) In particular, if $\hat{\beta}_n = \hat{\gamma}_n = 0$ for all $n \geq 0$, the $\{x_n\}$ defined by

$$x_0 \in K, \qquad x_{n+1} = \alpha_n x_n + \beta_n T x_n + \gamma_n u_n, \qquad n \geq 0,$$

is called the Mann iteration sequence with errors.

Remark 1. Note that the Ishikawa and Mann iterative processes are all special cases of the Ishikawa and Mann iterative processes with errors.

In 1994, Chidume proved a related result [5, Theorem 2] that deals with the Ishikawa iterative approximation of the fixed point of the class of Lipschitz strictly pseudo-contractive mappings, and put forth the following open problem.

Open Problem (Chidume [5]). It is not known whether or not the Ishikawa iteration sequence converges for a continuous strongly pseudo-contractive mapping.

The objective of this paper is (by using a new approximation technique, Lemma 1 (below)) to study convergence problems of Ishikawa and Mann iterative processes with errors for strongly pseudocontractive mappings and strongly accretive mappings.

2. LEMMAS

The following three lemmas play a crucial role in the proofs of our main result.

LEMMA 1. *Let X be a real uniformly smooth Banach space and let $J: X \to 2^{X^*}$ be the normalized duality mapping. Then for any $x, y \in X$ we have*

$$\|x + y\|^2 \le \|x\|^2 + 2(y, j), \qquad \forall j \in J(x + y). \tag{2.4}$$

Proof. We know that J can be equivalently defined as the subdifferential of the functional $\psi(x) = \frac{1}{2}\|x\|^2$ (see Asplund [1]), i.e.,

$$J(x) = \partial\psi(x) = \{f \in X^*: \psi(y) - \psi(x) \ge (f, y - x), \forall y \in X\}.$$

It follows from the definition of the subdifferential of ψ that

$$\psi(x) - \psi(x + y) \ge (j, x - (x + y)) = -(j, y), \qquad \forall j \in J(x + y).$$

Hence, we have that

$$\|x + y\|^2 \le \|x\|^2 + 2(y, j), \qquad \forall j \in J(x + y).$$

This completes the proof.

LEMMA 2 [3]. *X is a uniformly smooth Banach space if and only if J is single valued and uniformly continuous on any bounded subset of X.*

LEMMA 3 [12]. *Let a_n, b_n, and c_n be three nonnegative real sequences satisfying*

$$a_{n+1} \le (1 - t_n)a_n + b_n + c_n, \qquad n \ge 0, \tag{2.5}$$

with $t_n \in [0, 1]$, $\sum t_n = +\infty$, $b_n = O(t_n)$, and $\sum c_n < +\infty$. Then $\lim a_n = 0$.

3. MAIN RESULTS

THEOREM 3.1. *Let X be a real uniformly smooth Banach space and let $T: X \to X$ be a strongly accretive mapping. Define $S: X \to X$ by $Sx = x - Tx + f$,*

$\forall x \in X$, and suppose that the range of S is bounded. For arbitrary $x_0 \in X$ the Ishikawa iteration sequence $\{x_n\}$ with errors is defined by

$$x_{n+1} = \alpha_n x_n + \beta_n S y_n + \gamma_n u_n, \qquad y_n = \hat{\alpha}_n x_n + \hat{\beta}_n S x_n + \hat{\gamma}_n v_n, \qquad n \geq 0, \tag{3.6}$$

satisfying

(i) $\lim_{n\to\infty} \beta_n = 0$ *and* $\sum_{n=0}^{+\infty} \beta_n = +\infty$;

(ii) $\lim_{n\to\infty} \hat{\beta}_n = 0$;

(iii) $\lim_{n\to\infty} \hat{\gamma}_n = 0$ *and* $\sum_{n=0}^{+\infty} \gamma_n < +\infty$,

where $\alpha_n + \beta_n + \gamma_n = \hat{\alpha}_n + \hat{\beta}_n + \hat{\gamma}_n = 1$ $(n \geq 0)$, $\{u_n\}$ and $\{v_n\}$ are two bounded subsets of X. If there exists a solution of the equation $Tx = f$ for some $f \in X$, then, for arbitrary $x_0 \in X$, the sequence $\{x_n\}$ converges strongly to the unique solution of the equation $Tx = f$.

Proof. Let $Tq = f$, so that q is a fixed point of S. Since T is strongly accretive, there exists a $k \in (0,1)$ such that

$$(Tx - Ty, j(x-y)) \geq k\|x-y\|^2, \qquad \forall x, y \in X.$$

It follows from the definition of S that

$$(Sx - Sy, j(x-y)) \leq (1-k)\|x-y\|^2, \qquad \forall x, y \in X. \tag{3.7}$$

Since $\{u_n\}$ and $\{v_n\}$ are bounded, we can set

$$d = \max\left\{ \sup_{n\geq 0} \{\|u_n - q\|\}, \sup_{n\geq 0} \{\|v_n - q\|\}, \sup_{x\in X} \{\|Sx - q\|\}, \|x_0 - q\| \right\}. \tag{3.8}$$

By using (3.6), (1.3), (3.8), and induction, we obtain

$$\|x_n - q\| \leq d, \qquad \forall n \geq 0. \tag{3.9}$$

Moreover, by using (3.6), (1.3), (3.8), (3.9), (2.4), and (3.10), we have that

$$\|y_n - q\| \leq d, \qquad \forall n \geq 0,$$

and

$$\begin{aligned}\|y_n - q\|^2 &= \left\| \hat{\alpha}_n(x_n - q) + \hat{\beta}_n(Sx_n - q) + \hat{\gamma}_n(v_n - q) \right\|^2 \\ &\le \hat{\alpha}_n^2\|x_n - q\|^2 + 2\hat{\beta}_n\big(Sx_n - q, j(y_n - q)\big) \\ &\quad + 2\hat{\gamma}_n\big(v_n - q, j(y_n - q)\big) \\ &\le \hat{\alpha}_n^2\|x_n - q\|^2 + 2\|y_n - q\|\big(\hat{\beta}_n\|Sx_n - q\| + \hat{\gamma}_n\|v_n - q\|\big) \\ &\le \hat{\alpha}_n^2\|x_n - q\|^2 + 2d^2\big(\hat{\beta}_n + \hat{\gamma}_n\big), \qquad n \ge 0. \end{aligned} \tag{3.10}$$

By (3.6), (2.4), (3.7), (3.8), and (3.10), we also have that

$$\begin{aligned}\|x_{n+1} - q\|^2 &= \| \alpha_n(x_n - q) + \beta_n(Sy_n - q) + \gamma_n(u_n - q) \|^2 \\ &\le \alpha_n^2\|x_n - q\|^2 + 2\beta_n\big(Sy_n - q, j(x_{n+1} - q)\big) \\ &\quad + 2\gamma_n\big(u_n - q, j(x_{n+1} - q)\big) \\ &\le \alpha_n^2\|x_n - q\|^2 + 2\beta_n\big(Sy_n - q, j(y_n - q)\big) \\ &\quad + 2\beta_n\big(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)\big) \\ &\quad + 2\gamma_n\|u_n - q\|\,\| j(x_{n+1} - q)\| \\ &\le \alpha_n^2\|x_n - q\|^2 + 2\beta_n(1 - k)\|y_n - q\|^2 \\ &\quad + 2\beta_n\big(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)\big) + 2\gamma_n d^2 \\ &\le \alpha_n^2\|x_n - q\|^2 + 2(1 - k)\beta_n\hat{\alpha}_n^2\|x_n - q\|^2 \\ &\quad + 4(1 - k)\, d^2\beta_n\big(\hat{\beta}_n + \hat{\gamma}_n\big) \\ &\quad + 2\beta_n\big(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)\big) + 2\gamma_n d^2 \\ &\le A_n\|x_n - q\|^2 + b_n + c_n, \qquad n \ge 0, \end{aligned} \tag{3.11}$$

where

$$A_n = \alpha_n^2 + 2(1 - k)\beta_n\hat{\alpha}_n^2,$$

$$b_n = 4(1 - k)\, d^2\beta_n\big(\hat{\beta}_n + \hat{\gamma}_n\big) + 2\beta_n\big(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)\big),$$

$$c_n = 2\gamma_n d^2.$$

(1) First, we consider A_n. From (i), there exists a positive integer n_0 such that $\beta_n - k \le 0$ for all $n \ge n_0$. Therefore, we have that

$$\begin{aligned} A_n &\le (1 - \beta_n)^2 + 2(1 - k)\beta_n \\ &= 1 - k\beta_n + \beta_n(\beta_n - k) \le 1 - k\beta_n, \qquad \forall n \ge n_0. \end{aligned}$$

(2) Next, we prove that $(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)) \to 0$ (as $n \to \infty$). In fact, since

$$\begin{aligned}(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)) \\ &= (Sy_n - q, j(x_{n+1} - q) - j(x_n - q)) \\ &\quad + (Sy_n - q, j(x_n - q) - j(y_n - q)), \end{aligned} \tag{3.12}$$

$$\begin{aligned}\|x_{n+1} - q - (x_n - q)\| &= \|\beta_n(Sy_n - x_n) + \gamma_n(u_n - x_n)\| \\ &\le 2d(\beta_n + \gamma_n) \to 0 \qquad (\text{as } n \to \infty),\end{aligned}$$

$$\begin{aligned}\|x_n - q - (y_n - q)\| &= \|\hat{\beta}_n(x_n - Sx_n) + \hat{\gamma}_n(x_n - v_n)\| \\ &\le 2d\left(\hat{\beta}_n + \hat{\gamma}_n\right) \to 0 \qquad (\text{as } n \to \infty),\end{aligned}$$

$\{x_{n+1} - q, y_n - q\}$ and $\{Sy_n - q\}$ are bounded sets, and X is uniformly smooth so that j is uniformly continuous on any bounded subset of X, we have that

$$\|j(x_{n+1} - q) - j(x_n - q)\| \to 0 \qquad (\text{as } n \to \infty),$$

$$\|j(x_n - q) - j(y_n - q)\| \to 0 \qquad (\text{as } n \to \infty),$$

and

$$(Sy_n - q, j(x_{n+1} - q) - j(y_n - q)) \to 0 \qquad (\text{as } n \to \infty).$$

So, $b_n = O(k\beta_n)$.

(3) Finally, we know that c_n is summable.

Now, let $\|x_n - q\|^2 = a_n$ and $k\beta_n = t_n$ for each $n \ge n_0$. The inequality (3.11) reduces to

$$a_{n+1} \le (1 - t_n)a_n + b_n + c_n, \qquad n \ge n_0.$$

It follows from Lemma 2.3 that $x_n \to q \in X$. Suppose $q^* \in K$ is another fixed point of S. By (3.7), we have

$$\|q - q^*\|^2 = (Sq - Sq^*, j(q - q^*)) \le (1 - k)\|q - q^*\|^2.$$

Since $k \in (0, 1)$, we obtain the equality that $q = q^*$. This completes the proof.

Remark 2. Theorem 3.1 improves the main result of Liu (Theorem 1 of [12]) in the following ways:

(1) that $\{u_n\}$ and $\{v_n\}$ be two summable sequences is replaced by $\{u_n\}$ and $\{v_n\}$ be two bounded sequences;

(2) T may not be Lipschitz;

(3) Condition (ii) is weaker than condition (ii) in Theorem 1 of Liu [12].

If T is demicontinuous (i.e., $x_n \xrightarrow{s} x \Rightarrow Tx_n \xrightarrow{w} Tx$), then, for each $f \in X$, the existence of the solution of the equation $Tx = f$ follows from Deimling [6].

In Theorem 3.1, if $\hat{\beta}_n = \hat{\gamma}_n = 0$ for all $n \geq 0$, then we obtain a result that deals with the Mann iterative process with errors as follows.

COROLLARY 3.2. *Let X be a real uniformly smooth Banach space and let T: $X \to X$ be a demicontinuous and strongly accretive mapping. Define S: $X \to X$ by $Sx = x - Tx + f$, $\forall x \in X$. For arbitrary $x_0 \in X$, the Mann iteration sequence $\{x_n\}$ with errors is defined by*

$$x_{n+1} = \alpha_n x_n + \beta_n Sx_n + \gamma_n u_n, \qquad n \geq 0,$$

satisfying

(i) $\lim_{n\to\infty} \beta_n = 0$ *and* $\sum_{n=0}^{+\infty} \beta_n = +\infty$,

(ii) $\sum_{n=0}^{+\infty} \gamma_n < +\infty$,

where $\alpha_n + \beta_n + \gamma_n = 1$ $(n \geq 0)$ and $\{u_n\}$ is a bounded subset of X. If the sequence $\{Sx_n\}$ is bounded in X, then the Mann iterative sequence $\{x_n\}$ converges strongly to the unique solution $q \in X$ of the equation $Tx = f$.

Remark 3. Note that the assumption of the range of S in Theorem 3.1 is replaced by $\{Sx_n\}$ is bounded.

Now, we turn to study the Ishikawa and Mann iterative process with errors for the strongly pseudo-contractive mapping.

THEOREM 3.3. *Let X be a real uniformly smooth Banach space, let K be a nonempty bounded closed convex subset of X, and let T: $K \to K$ be a strongly pseudo-contractive mapping. Let q be a fixed point of T and let the Ishikawa iteration sequence $\{x_n\}$ be defined by*

$$\begin{aligned} &x_0 \in K, \\ &x_{n+1} = \alpha_n x_n + \beta_n Ty_n + \gamma_n u_n, \\ &y_n = \hat{\alpha}_n x_n + \hat{\beta}_n Tx_n + \hat{\gamma}_n v_n, \qquad n \geq 0, \end{aligned} \tag{3.13}$$

satisfying

(i) $\lim_{n\to\infty} \beta_n = 0$ *and* $\sum_{n=0}^{+\infty} \beta_n = +\infty$;

(ii) $\lim_{n\to\infty} \hat{\beta}_n = 0$;

(iii) $\lim_{n\to\infty} \hat{\gamma}_n = 0$ *and* $\sum_{n=0}^{+\infty} \gamma_n < +\infty$.

Then $\{x_n\}$ *converges strongly to the unique fixed point of* T.

Proof. Obviously, $\{x_n\}$ and $\{y_n\}$ are both contained in K. Since T is strongly pseudo-contractive, $I - T$ is strongly accretive. From Definition 1(i), for any given $x, y \in K$ we have

$$(Tx - Ty, j(x-y)) \le (1-k)\|x-y\|^2,$$

where the constant $k \in (0,1)$. By using (3.13), (2.4), (3.8), (3.12), and (3.10), we have that

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \|\alpha_n(x_n - q) + \beta_n(Ty_n - q) + \gamma_n(u_n - q)\|^2 \\
&\le \alpha_n^2\|x_n - q\|^2 + 2\beta_n\big(Ty_n - q, j(x_{n+1} - q)\big) \\
&\quad + 2\gamma_n\big(u_n - q, j(x_{n+1} - q)\big) \\
&\le \alpha_n^2\|x_n - q\|^2 + 2\beta_n\big(Ty_n - q, j(y_n - q)\big) \\
&\quad + 2\beta_n\big(Ty_n - q, j(x_{n+1} - q) - j(y_n - q)\big) \\
&\quad + 2\gamma_n\|u_n - q\|\|j(x_{n+1} - q)\| \\
&\le \left[\alpha_n^2 + 2(1-k)\beta_n\hat{\alpha}_n^2\right]\|x_n - q\|^2 \\
&\quad + 4(1-k)\,d^2\beta_n\big(\hat{\beta}_n + \hat{\gamma}_n\big) + 2\beta_n e_n + 2\,d^2\gamma_n, \qquad n \ge 0,
\end{aligned} \tag{3.14}$$

where $e_n = (Ty_n - q, j(x_{n+1} - q) - j(y_n - q))$. From (3.12) and similar arguments as in the proof of Theorem 3.1, we have that $e_n \to 0$ (as $n \to 0$). The inequality (3.14) reduces to

$$\|x_{n+1} - q\|^2 \le (1 - k\beta_n)\|x_n - q\|^2 + b_n + c_n, \qquad n \ge n_0,$$

where

$$\begin{aligned}
b_n &= 4(1-k)\,d^2\beta_n\big(\hat{\beta}_n + \hat{\gamma}_n\big) + 2\beta_n e_n = O(k\beta_n), \\
c_n &= 2\gamma_n d^2.
\end{aligned}$$

Applying Lemma 2.3, we can obtain that $\{x_n\}$ converges strongly to $q \in K$. In the same way as in the proof of Theorem 3.1, we can prove that q is the unique fixed point of T.

Remark 4. If T is a continuous mapping in Theorem 3.3, then T has a fixed point by Proposition 3 of Martin [14]. So, it not only gives an affirmative answer to the open problem (Chidume [5]) in the more general setting, but also the corresponding results in Chidume [4, 5], Liu [12], Deng [7–9], and Zhou [16] are all special cases of Theorem 3.3. By the way, Theorem 3.3 also corrects a mistake in Theorem 2 of Liu [12].

COROLLARY 3.4. *Let K be a nonempty bounded closed convex subset of X of a real uniformly smooth Banach space X. Let $T: K \to K$ be a strongly pseudo-contractive mapping. Let q be a fixed point of T and let the Mann iteration sequence $\{x_n\}$ be defined by*

$$\begin{aligned} &x_0 \in K, \\ &x_{n+1} = \alpha_n x_n + \beta_n T x_n + \gamma_n u_n, \qquad n \geq 0, \end{aligned}$$

satisfying

(i) $\lim_{n \to \infty} \beta_n = 0$ *and* $\sum_{n=0}^{+\infty} \beta_n = +\infty$;

(ii) $\sum_{n=0}^{+\infty} \gamma_n < +\infty$.

Then $\{x_n\}$ converges strongly to the unique fixed point of T.

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